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The orientation of the Hertzian cone crack in soda-lime glass formed by oblique dynamic and quasi-static loading with a hard sphere

M. MUNAWAR CHAUDHRI, CHEN LIANGYI*
Physics and Chemistry of Solids, Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, UK

High speed photographic studies, made at a framing speed of 10^6 frames per second, of the oblique impact of 2 mm diameter tungsten carbide spheres on blocks of soda-lime glass are described. The oblique impact causes sliding, which has a very marked influence on the orientation of the Hertzian cone crack with respect to the surface normal. An explanation of this effect is also provided. Similar observations are made from oblique quasi-static loading experiments.

1. Introduction

One of the major causes of strength degradation of brittle materials, such as the silicate glasses and ceramics, during service and handling, is the generation of damage, usually in the form of microcracks, due to contact. In order to improve the resistance of these materials to the formation of such damage, it is important to understand the damage mechanisms involved. It may be said that no matter how carefully the inherent flaws of a brittle sample may have been removed during its manufacture, once contact damage occurs, most of the effort put into manufacturing the flaw-free sample effectively goes to waste. In fact, if it were not due to the limitations imposed by any potential contact damage, we would see much more practical use and applications of brittle materials than is the case at present.

Contact damage can occur due to both static and impact loading. In a series of investigations [1–9] we have shown by high-speed photography the sequence of formation of various types of crack in a number of brittle glasses and ionic crystals during impact loading and unloading with small spherical or pointed projectiles. Other workers [10] have correlated the loss of strength of glass specimens with projectile impact parameters.

In the previous studies, the impact loading was normal to the specimen surface. There has, however, been little work done on the impact loading situation in which both the normal and tangential (i.e. parallel to the surface) forces are applied simultaneously. In some respects this problem has some similarities with the one in which a slider (e.g. a sphere), which is statically loaded normally on to a brittle surface, is slid along the surface due to the application of a tangential force. In the latter case, it has been found by several researchers [11–13] that the value of the critical normal load on a spherical indentor required to cause the formation of the Hertzian cone crack is considerably smaller than that in the absence of the tangential force. A number of authors have treated the problem theoretically [14–16]; the main conclusion is that at the trailing edge of the contact between the sliding sphere and the specimen, enhanced tensile stresses are generated, where cone crack initiation occurs. One very interesting feature of such cracks is that they are only "partial" cones.

Recently, a simplified view of the formation of the partial cone cracks has been put forward by Lawn *et al.* [15]. According to them, the effects of the tangential loading are to increase the magnitude of the resultant load and to alter its direction with respect to the surface normal. Under critical loading conditions, the Hertzian cone crack that forms has its axis oriented in the direction of the resultant load, but its included angle remains unaltered. For a sufficiently large change in the direction of the resultant load, a part of the cone crack may protrude out of the surface, thus giving rise to a partial cone crack.

If μ is the coefficient of friction between the sphere and the flat, and W the normal load on the former, the magnitude of the resultant load, W_R, is given by

$$W_R^2 = W^2 + \mu^2 W^2 \qquad (1)$$

The angle, β, between the surface normal and the resultant load direction is given by

$$\tan\beta = \left(\frac{\mu W}{W}\right) = \mu \qquad (2)$$

In the case of a steel sphere sliding on glass, $\mu = 0.4$. Therefore, $\beta = \tan^{-1}0.4 \approx 22°$. In soda-lime glass, the included angle of the Hertzian cone [17] crack is $\sim 136°$. Therefore in the case of a steel sphere sliding on a soda-lime surface, we would expect to have only

*Present address: Xian Institute of Optics and Precision Mechanics, Xian, China.

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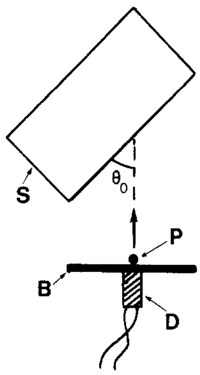

Figure 1 Schematic diagram of the experimental arrangement used for dynamic loading. S, soda-lime glass block; P, 2 mm diameter tungsten carbide projectile; B, 0.6 mm thick brass plate; D, fast acting electric detonator.

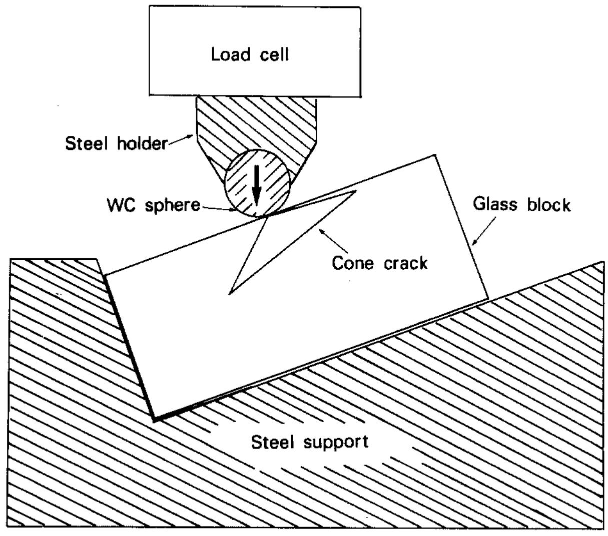

Figure 2 Schematic diagram of the experimental arrangement used for oblique quasi-static loading.

a partial cone crack. This prediction is apparently supported by previous experiments.

In impact, the simultaneous application of normal and tangential loading can occur for the following reasons. Consider that the impact direction is tilted at an angle θ_0 to the surface (see Fig. 1). Thus for an impact velocity v, the components along the surface normal and parallel to the surface are $v \sin \theta_0$ and $v \cos \theta_0$, respectively. Initially, the impacting sphere is not spinning and therefore on impact it will slide on the surface due to the velocity component $v \cos \theta_0$. Also, the frictional forces will impart an angular velocity to the sphere about an horizontal axis parallel to the impact surface. The sliding of the sphere will continue until the linear circumferential velocity equals $\sim v \cos \theta_0$. The frictional force imparted to the surface is in the direction of sliding.

From the above it is clear that the impact of a sphere on an inclined surface gives rise to simultaneous normal and tangential loading. It was the purpose of the studies, which are reported below, to examine the effects of such a loading on the damage morphology in soda-lime glass and its chronological order. Some of the results from these studies were recently reported at a meeting [18]. It is found that the orientation of the damage (i.e. Hertzian cone crack) from both the dynamic and quasi-static experiments is completely opposite to the results obtained from the quasi-static sliding experiments of Lawn *et al.* [15]. An explanation of these differences is also presented.

2. Experimental procedure

The soda-lime glass blocks used for this work were of size 40 mm × 40 mm × 25 mm. The larger faces of the blocks were in the as-received transparent state, while the smaller faces were polished down to an optical finish; the impact studies were made on these faces. The projectiles were 2 mm diameter tungsten carbide (WC) spheres, which were propelled vertically upwards towards the target using an explosive gun at a velocity of $\sim 150\,\text{m}\,\text{sec}^{-1}$. Employing a specially designed target holder, the angle between the impact direction and the target surface could be varied between 15° and 90°. A schematic diagram of the projectile and target is shown in Fig. 1. The initiation and growth of the impact damage were photographed at a framing speed of 10^6 frames per second with a Beckman and Whitley Model 189 rotating mirror framing camera using back lighting. The photographic film used was Ilford HP5, which was developed in ID11 for 7 to 8 min at room temperature. In a few experiments, the photographic investigations were made using circularly polarized light, which helped us visualize the nature of the stress field around an impact site.

In order to investigate further the influence of friction between the projectile and the target, the latter was coated with a thin film of lubricating grease.

It may be noted from the photo-graphic sequences, which are presented in Section 3, that the projectile appears to be moving vertically downwards. This change of orientation by 180° was done in order to improve the presentation of the photographic sequences.

A schematic diagram of the arrangement used for making the quasi-static experiments is shown in Fig. 2. The test specimen was placed on a rigid steel support which was bolted down firmly on to the base of an Instron model 1122 mechanical testing machine. The glass specimen was loaded with a 2 mm diameter tungsten carbide sphere moving at a speed of 0.05 mm min^{-1}. The sphere was firmly held in a steel holder, which in turn was screwed into a load cell mounted on the cross-head of the machine. The damage and the stress pattern within the specimen were viewed along a direction normal to the load axis using circularly polarized light arranged to reveal the isochromatics within the loaded glass specimen.

3. Results

Typical high-speed photographic sequences for different angles of impact θ_0, are presented in Figs 3 to 6. It will be observed from the sequences that as the angle of impact decreases from 90°, that between the axis of the cone crack and the normal to the surface increases systematically. This tilting of the cone is such that its front (i.e., towards the impact direction surface makes a significantly greater angle with the impact surface than does the rear one. Also, the degree of tilt changes during the contact time of the projectile.

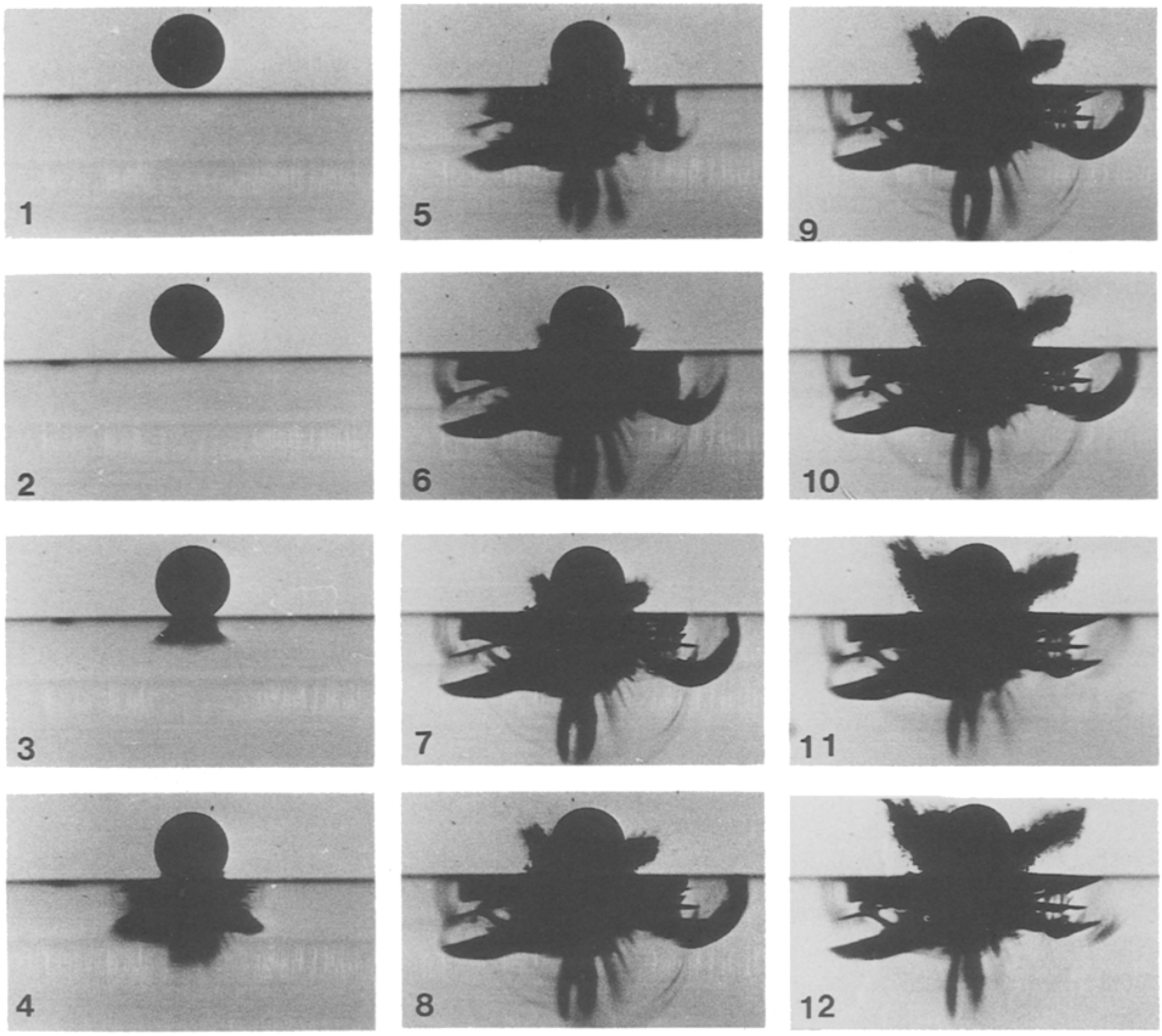

Figure 3 Impact of a 2 mm diameter tungsten carbide sphere on a block of soda-lime glass. Impact velocity 130 m sec^{-1}; impact angle 90°; interframe time 1 μ sec.

For example, consider the case of the 20° impact (see Fig. 6). The projectile makes contact with the glass between frames 3 and 4, resulting in the formation of a cone of included angle of 118°; the values of the angles between the front and the rear sides of the cone with the impact surface are 36° and 26°, respectively. The cone crack velocity between frames 3 and 4 is at least 1200 m sec^{-1}. In frame 5, a median crack, M, has formed, and its velocity is at least 1870 m sec^{-1}. Note also that during this 1 μ sec period the cone crack has extended in such a way that the angle between its rear surface and the impact surface has decreased, but that between the front surface of the cone and the impact surface has increased. A consequence of this is that the apparent included angle of the cone increases from 118° in frame 4 to 123° in frame 5. This process of the change of the included cone angle and its tilt continues in frame 6, where the value of the former has become 135°. Note also that the rear surface of the cone crack has now become parallel to the impact surface. Between frames 5 and 6, the cone has extended further at a velocity of $\sim$ 270 m sec^{-1}, whereas the median crack propagates at $\sim$ 135 m sec^{-1}. After frame 6, there is only a very slight increase in the size of the cone crack, but there is no growth in the extent of the median crack. An important observation is that during the contact time, the impacting sphere has slid on the glass surface at $\sim$ 100 to 120 m sec^{-1}.

Median cracks form for all angles of impact studied in these experiments. However, their size as measured along the normal to the impact surface, was the largest for the 90° impact and smallest when the impact angle was 15°. Lateral cracks also formed during unloading. The total contact time was in the range 4 to 10 μ sec. In all cases, except when $\theta_0 \approx 90°$, the rebound angle was smaller than the incidence angle.

Collective presentation of the impact damage during the early stage of its formation and its final form at the end of the impact is shown in Figs 7 and 8, respectively. From these figures it can be readily seen that the nature and orientation of the damage change considerably during the contact time.

A sequence of photographs showing the isochromatic fringes produced due to the impact with a WC sphere is presented in Fig. 9. Here the sphere impacts the glass at an angle of 25° between frames 5 and 6. The isochromatic fringes are clearly visible in frame 6; within each fringe the magnitude of the shear stress is constant. It is interesting to note that the fringes are

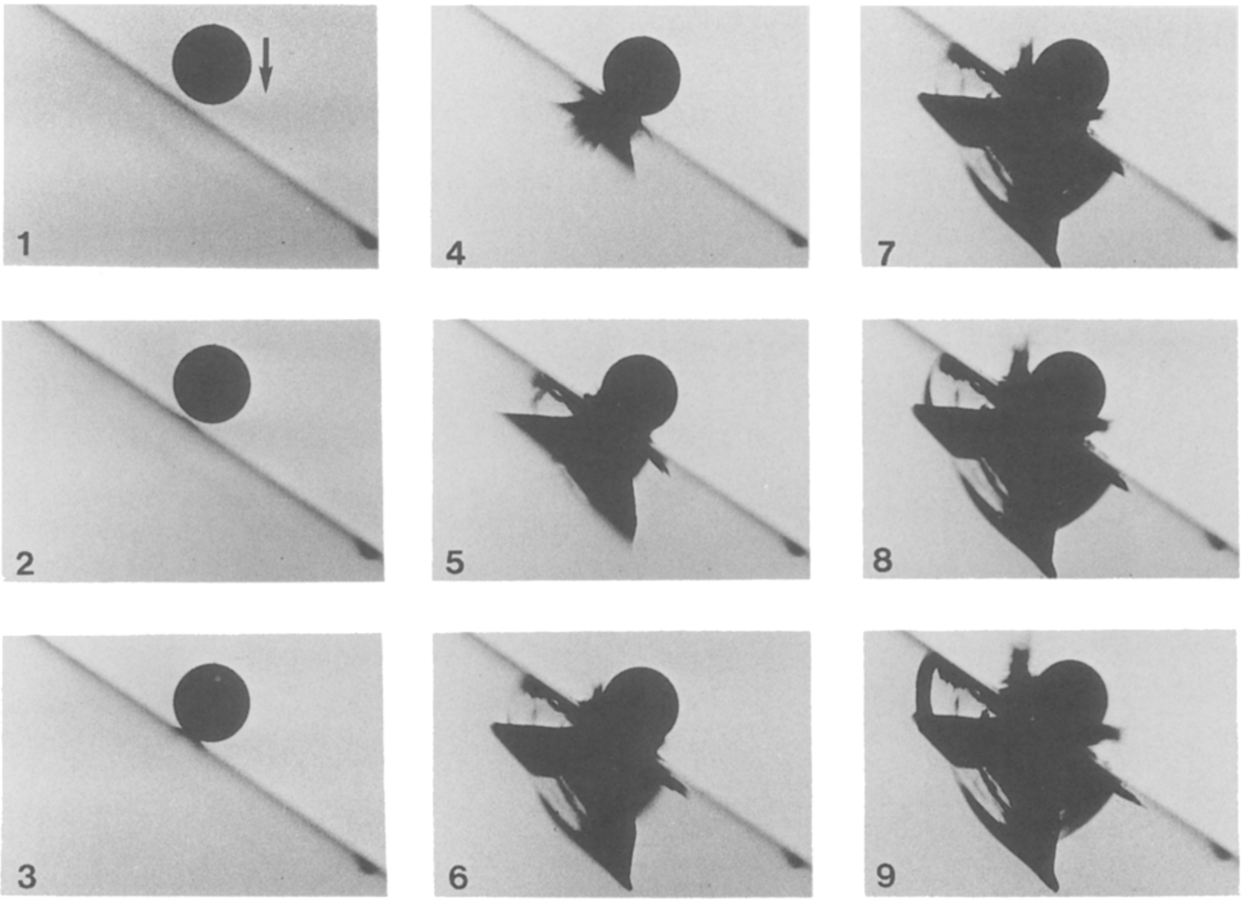

Figure 4 Impact of a 2 mm diameter tungsten carbide sphere on a block of soda-lime glass. Impact velocity 125 m sec^{-1}; impact angle 55°; interframe time 1 μsec.

oriented symmetrically to the surface normal. This was also the case when the impact surface had been lubricated with grease (sequence not shown).

In the quasi-static experiments as well, the axis of the Hertzian cone crack was similarly tilted with respect to the specimen surface normal. An example is shown in Fig. 10. Here the angle between the surface normal and the vertical is 20°. In this case, as the indentor load increases, the centre of contact between the sphere and the glass flat moves down along the inclined surface, which we define as the forward direction. It will be seen from Fig. 10 that the front part of the cone crack makes an angle of 44° with the indented surface, whereas its rear part coincides with the latter (compare with Fig 6). The included angle of the cone is 136°, which is approximately the same as that of the Hertzian cone crack in the glass produced by normal loading.

4. Discussion

The high-speed photographic sequences have shown that for the angle of impact in the range 15° to 90°, complete Hertzian cone cracks form. This observation is different from the results obtained from the sliding of a sphere normally loaded quasi-statically on a glass surface; in this case only partial cone cracks are formed. Under conditions of quasi-static sliding of a steel or WC sphere on a smooth glass surface, the coefficient of friction [13] is about 0.4. Therefore, using this value for our impact experiments and Equation 2, we would expect the resultant load axis to be tilted away from the normal and towards the front side of the impact. The same orientation would be expected for the cone axis if the Lawn *et al.* [15] proposal were correct. In the impact experiments, the results are just the opposite of this. We suggest that the reason for this is the displacement of the contact during the growth of the Hertzian cone crack. Within this period, the crack growth occurs in a rapidly changing stress field. If a is the radius of the surface ring crack (assumed to be located at the edge of the contact), then the front side of the cone crack grows along the principal stress trajectories for which a_{cf}/a becomes increasingly smaller than 1, where a_{cf} is the distance between the front side of the ring crack and the centre of the sliding contact. On the other hand, the rear side of the cone crack grows along the principal stress trajectories for which a_{cr}/a becomes increasingly greater than 1 with increasing time, where a_{cr} is the distance between the rear side of the ring crack and the centre of the sliding contact. Because the angle between a principal stress trajectory and the specimen surface decreases with increasing r/a, the dimensionless starting distance of a principal stress trajectory, it is suggested that the front side of the cone will make a greater angle with the surface than the rear one. This is in agreement with our observations. Moreover, because the sliding is initially at a velocity $\sim v \cos \theta_0$, the above effect will be greater for a smaller θ_0.

There is another factor which comes into play

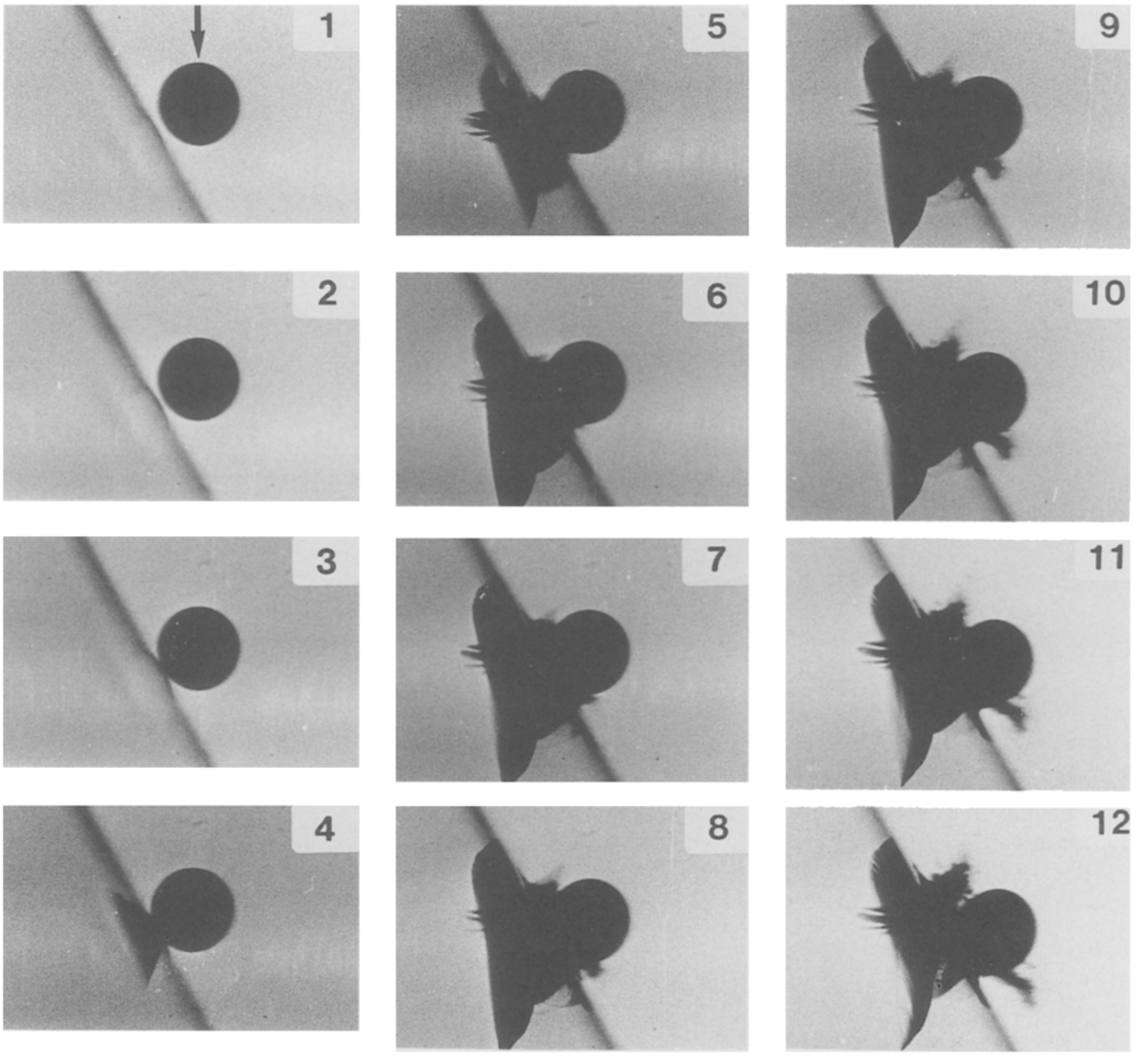

Figure 5 Impact of a 2 mm diameter tungsten carbide sphere on a block of soda-lime glass. Impact velocity 150 m sec^{-1}; impact angle 30°; interframe time 1 μsec.

during an oblique impact. The normal component of the impact velocity is $v \sin \theta_0$. Therefore, even if there were no sliding then, following the early observations from normal impact experiments [6], we would expect the included angle of the cone to decrease with increasing θ_0. This is also borne out by the measured included angles during the very early stages of their formation (see Fig. 7), when the amount of sliding will only be very small.

Similar arguments will also apply in the case of the oblique quasi-static loading.

As stated in Section 1, during an oblique impact, sliding of the sphere on the glass occurs, thus giving rise to a tangential stress of magnitude $\mu\sigma_n$, where σ_n is the normal stress. The fact that in the impact experiments the isochromatics are very nearly symmetrical about the surface normal before the formation of the cone crack (see Fig. 9) may be due to the following reasons.

1. For a very small value of μ, which can occur in an impact experiment [19], the tangential stress will be too small to affect significantly the symmetry of the isochromatics.

2. The influence of the tangential stress may be confined to a narrow region close to the surface, without influencing the distant field.

Reason 2 seems more plausible, as in the quasi-static loading in which $\mu = 0.4$, the isochromatics are also symmetrical about the surface normal before the formation of the cone crack.

The size of cone and median cracks is directly related to the magnitude of the dynamic load, which in turn is dependent on $v \sin \theta_0$. Our experimental results support this qualitatively.

5. Conclusions

Our high-speed framing photography experiments have shown that during the oblique impact of a tungsten carbide sphere on a brittle surface, such as glass, the phenomenon of sliding is very important in determining the included angle of the Hertzian cone crack and its orientation with respect to the surface normal. The orientation of the cone crack is completely opposite to that suggested by Lawn *et al.* [15]: the front part of the cone crack makes a significantly larger angle with the impact surface than does the rear

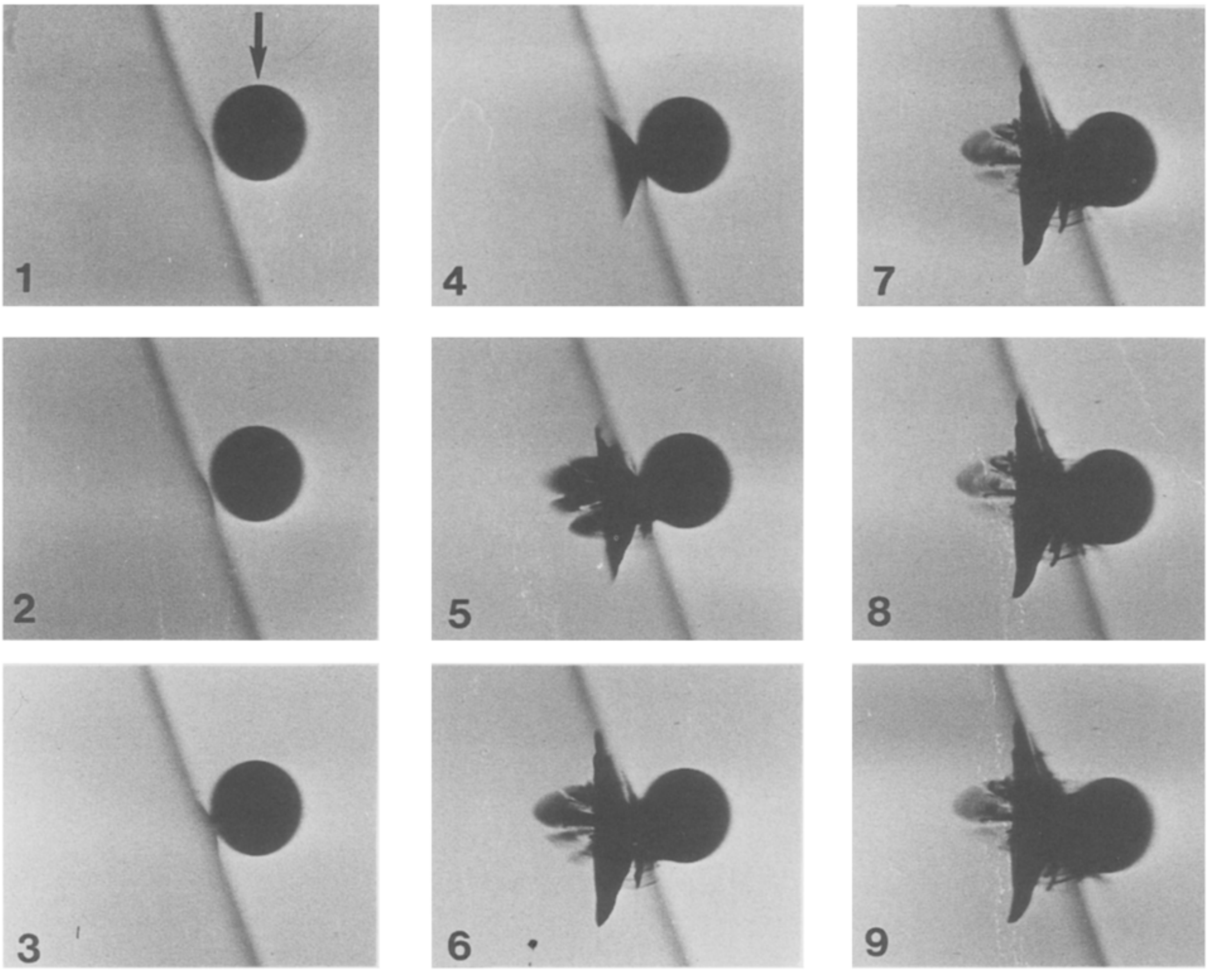

Figure 6 Impact of a 2 mm diameter tungsten carbide sphere on a block of soda-lime glass. Impact velocity 145 m sec^{-1}; impact angle 20°; interframe time 1 μsec. M (frame 5) is a median crack.

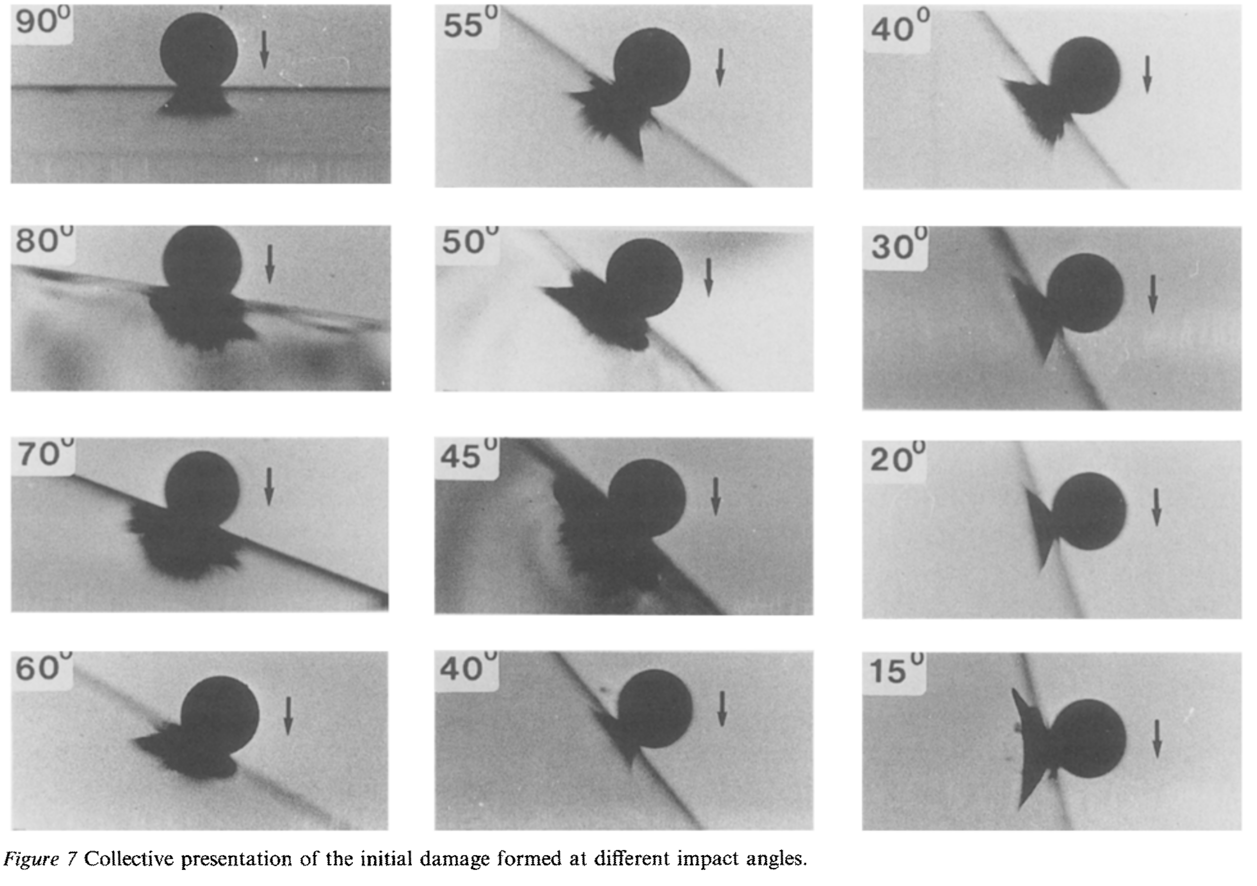

Figure 7 Collective presentation of the initial damage formed at different impact angles.

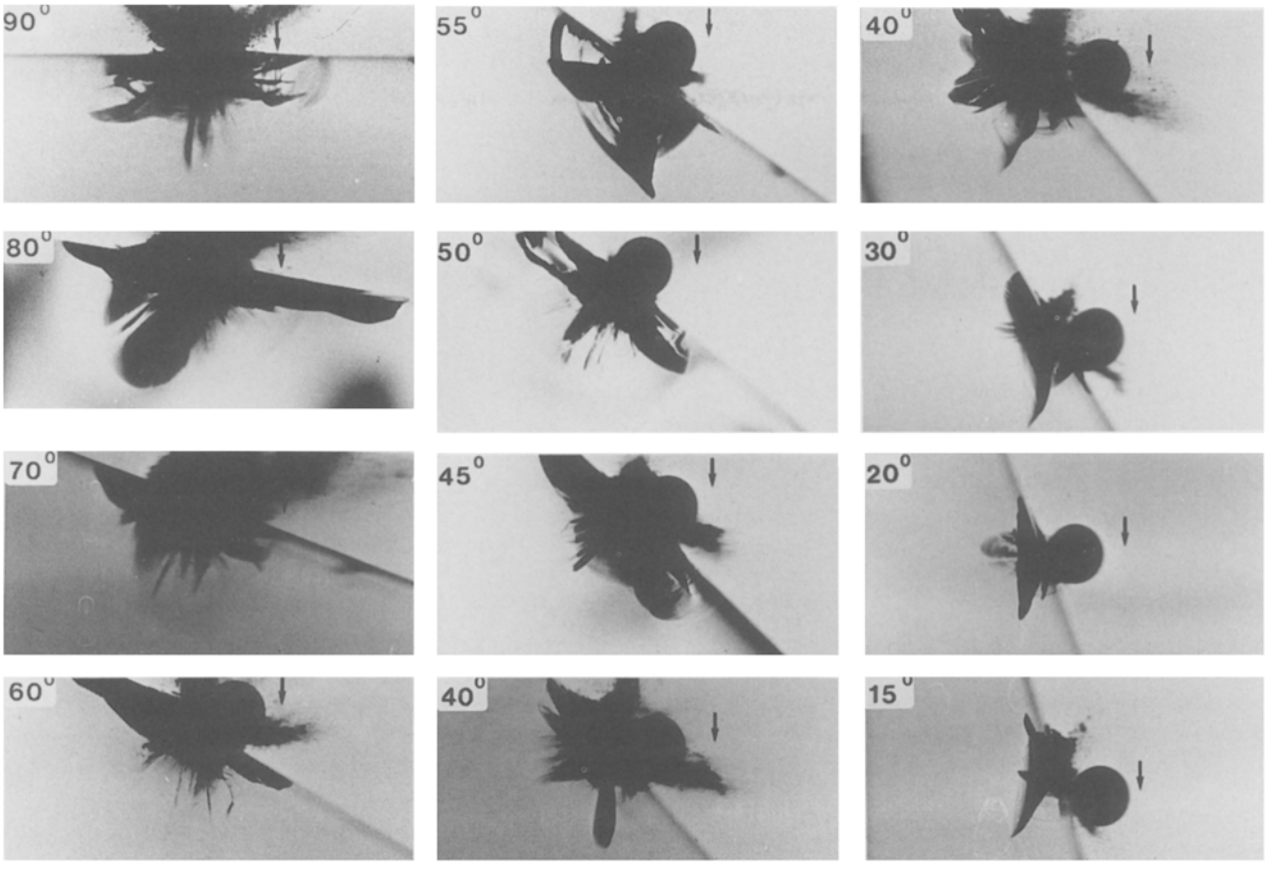

Figure 8 Collective presentation of the final damage formed at different impact angles.

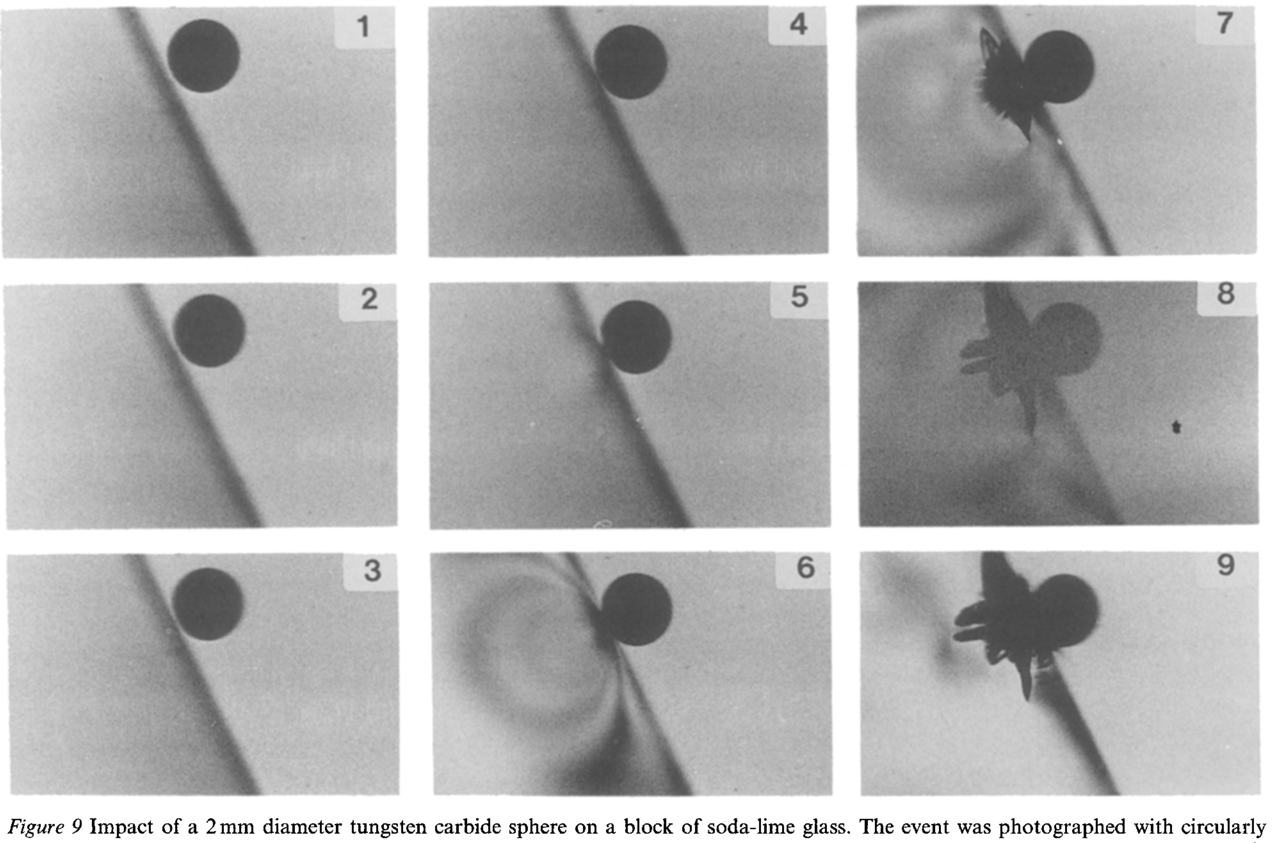

Figure 9 Impact of a 2 mm diameter tungsten carbide sphere on a block of soda-lime glass. The event was photographed with circularly polarized light. Note that the isochromatic fringes in frame 6 are symmetrical to the specimen surface normal. Impact velocity 125 m sec^{-1}; impact angle 25°; interframe time 1 μsec.

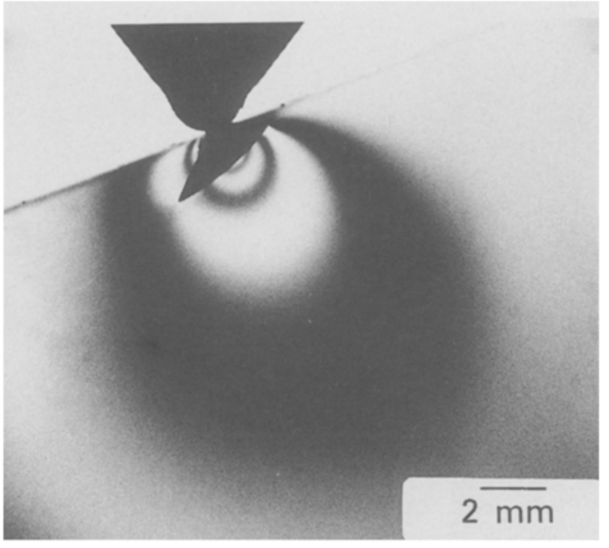

Figure 10 The orientation of the Hertzian cone crack in soda-lime glass due to oblique quasi-static loading with a 2 mm diameter tungsten carbide sphere. The angle between the load axis and the specimen surface normal: 20°; indentor load 90 kgf. Note that the front side of the cone makes an angle of 44° with the indented surface whereas the rear side coincides with it.

one. Similar results have also been obtained from quasi-static experiments. These observations have been explained by the displacement of the centre of contact during the crack growth.

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